The Harmonic Overtone Series

BY ROGER GOODMAN

In the last issue (see page 4, vol.4, no.6 in the Archives at www.FolkWorks.org) we looked at some examples of wind instruments and saw that their lexicon of notes are determined by how the standing waves are allowed to form within their enclosed columns of air. This time we will see how the same process works for stringed instruments and how this leads into another important concept, the harmonic overtone series.

QUICK REVIEW: Remember that any note that resonates from a musical instrument is a result of a standing wave. Standing waves are a result of the constructive interaction of reflected waves (see **Figure 1**). Standing waves have points called nodes where there is no displacement. This makes the wave appear to stand still. The boundary conditions (end points) of an instrument dictate where the nodes are allowed to form. In the case of the flute and clarinet the ends could be either open or closed. If closed then only a node can form there. If open then only an anti-node (a point of maximum displacement) can form there.

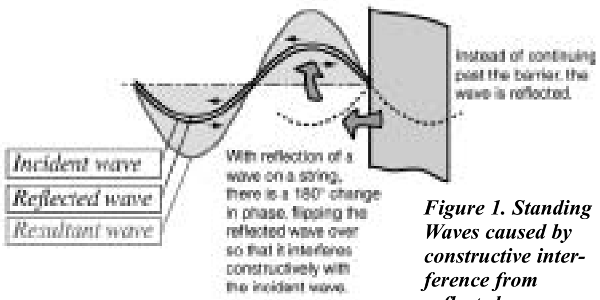

Figure 1. Standing Waves caused by constructive interference from reflected waves.

The situation with stringed instruments is actually a bit simpler since strings have only closed ends. This means that the ends of the strings, closed by the bridge at one end and the nut at the other, will always be nodes. When an open guitar string is plucked the note produced is that of the string vibrating in its fundamental mode. The fundamental mode of vibration for a string places the maximum movement (an anti-node) mid-way between the nodes at each end. In this mode the length of the string represents only half of the wavelength for the sounded note. An examination of the first illustration in Figure 2, where the dark vertical areas represent the closed ends of the string, should help to clarify this situation.

A guitar string can be coaxed into its next mode of vibration by forcing a node at the string's midway point. This is done by placing the pad of your finger tip lightly on the string just over the 12th fret. Pluck the string as usual and then remove the finger when the note begins to sound. You should hear a wonderfully clear, bell-like tone an octave above the pitch of the open string. This is the string's 2nd harmonic or the 1st overtone above the fundamental note. The second illustration in Figure 2 should help to visualize this. You can get the full set of harmonic overtones by successively dividing the string into thirds (at the 7th fret), fourths (at the 5th fret), and so on as shown in the last two illustrations in **Figure 2**.

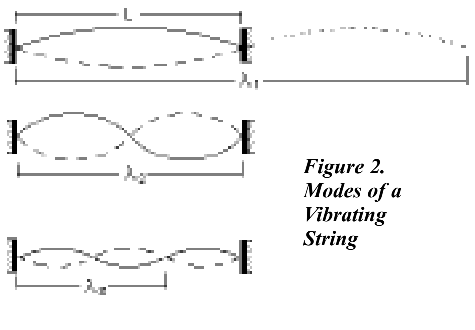

Figure 2. Modes of a Vibrating String

NOMENCLATURE: HARMONIC VS. OVERTONE.

There is some confusion about the use of the terms, "harmonic" and "overtone." Overtone should mean a tone over or above the fundamental but some authors use the two terms interchangeably, e.g. the 3rd harmonic would be the same as the 3rd overtone. A more accepted usage would be to name the fundamental as the 1st harmonic and the 2nd harmonic as the 1st overtone. This means that the 3rd harmonic would be the 2nd overtone. It has been my experience that this "off-by-one" numbering system, though arguably more correct, tends to be confusing. For that reason it will not be used in the remainder of this discussion.

Harmonic	Freq.	Note	Relation to Fundamental (Octave + Scale Degree)
1	110	A	Fundamental (Unison)
2	220	A	Octave 1
3	330	E	01 + 5
4	440	A	Octave 2
5	550	C#	02 + 3rd
6	660	E	02 + 5th
7	770	G#	02 + 7b(-)
8	880	A	Octave 3
9	990	B	03 + 2nd
10	1100	C#	03 + 3rd
11	1210	D	03 + 4th(+)
12	1320	E	03 + 5th
13	1430	F#	03 + 6( )
14	1540	G	03 + 7b
15	1650	G#	03 + 7(-)
16	1760	A	Octave 4

Figure 3. The first sixteen harmonics of the guitar's A string.

Now, let's examine the harmonics on the A (5th) string of the guitar. The open A string vibrates at 110 Hz. Playing the harmonic at the 12th fret divides the string in half which doubles the frequency to 220 Hz. That's another A but an octave higher than the fundamental. Playing the harmonic at the 7th fret divides the string into thirds producing triple the original frequency or 330 Hz, an E. Playing the harmonic at the 5th fret divides the string into fourths producing four times the original frequency or 440 Hz, yet another A, but now two octaves above the fundamental. Thus, each successive harmonic produces the next whole-number multiple of the fundamental frequency — Harmonic 1 = 1x110Hz, 2 = 2x110Hz, 3 = 3x110Hz, 4 = 4x110Hz and so on (see **Figure 3**). Notice that each octave of A is twice the frequency of the previous A and that each successive octave has twice the number of harmonics as the previous one.

This is the essential mechanics of the harmonic overtone series. Right about now you may be saying to yourself, "So what?" Well, for starters, let me say that the harmonic overtone series is a very important musical concept, just as important as our old friend the circle of fifths and perhaps in a more basic sense. The next installment will continue the discussion of the harmonic overtone series and some of its applications. Until then, be sure to notice the richness of the overtones in your own life and, of course, stay tuned.

Roger Goodman is a musician, mathematician, punster, reader of esoteric books and sometime writer, none of which pays the mortgage. For that, he is a computer network guy for a law firm. He has been part of the Los Angeles old-time & contradance music community for over thirty years. While not a dancer, he does play fiddle, guitar, harmonica, mandolin, banjo & spoons. Roger has a penchant for trivia and obscura and sometimes tries to explain how the clock works when asked only for the time. He lives with his wife, Monika White, in Santa Monica, CA.