VISUALIZING HARMONIES THROUGH THE BEAUTY OF LISSAJOUS CURVES

BY ROGER GOODMAN

What makes some musical intervals pleasing (consonant) while others are just plain annoying (dissonant)? Why do octaves, fifths, fourths and thirds (in that order) sound best to our ears and other intervals less so? What's the secret? It's not complicated—really—because the simpler the relationship between two notes, the more pleasing the interval sounds. This relationship could be visually demonstrated if we could draw pictures that represent the consonance of musical intervals. In this article, we will see that this is actually possible. And, it will not surprise regular readers of this column to learn that there is a mathematical relationship hiding within these musical questions and that there is a mathematical way to determine how pleasing a musical interval will be.

Let's start with the math. The most pleasant intervals can be expressed as the ratio of small whole numbers. If you double the frequency of a note (say, A-440) you get the same note name but an octave up (A-880). So the ratio here is two to one (2:1). This is about the simplest ratio you can get except for the unison (1:1). The simplest ratios are shown in Table 1.

Table 1 — *The Simplest Ratios*

Interval	Ratio	Notes & Frequencies	
Octave	2:1	A-880	A-440
Fifth	3:2	E-659.26	A-440
Fourth	4:3	D-587.33	A-440
Major Third	5:4	C#-554.37	A-440
Minor Third	6:5	C-523.25	A-440

The octave, fifth and fourth are the intervals which have been considered to be consonant throughout history by essentially all cultures. The ability to visualize consonance patterns is possible through the work of Jules Antoine Lissajous (1822-1880), a French mathematician who was interested in studying waves. His studies resulted in the beautiful family of Lissajous Curves (see **Figure 1**). Dissatisfied with the problems introduced by the existing methods of detection and measurement, Lissajous began by using a tuning fork to produce waves in water. In 1855 he described a way of studying acoustic vibrations by having a mirror attached to a vibrating object and reflecting a light beam from the mirror onto a screen. He was then able to devise a method of calibrating tuning forks, using two such mirrored forks (**see Figure 2**), one of a known frequency and another that was to be matched to the first. He would have the light beam reflect from a mirror on one of the tuning forks and fall onto the mirror of the other while it was vibrating at a right angle to the first. The resulting beam, now vibrating in two axes, would fall on a screen and the beautiful patterns that would come to be known as Lissajous Curves would appear. The patterns that appear are indicative of the ratio of these two frequencies. Today an oscilloscope is used to see these Lissajous figures which continue to be a common method of comparative frequency measurement.

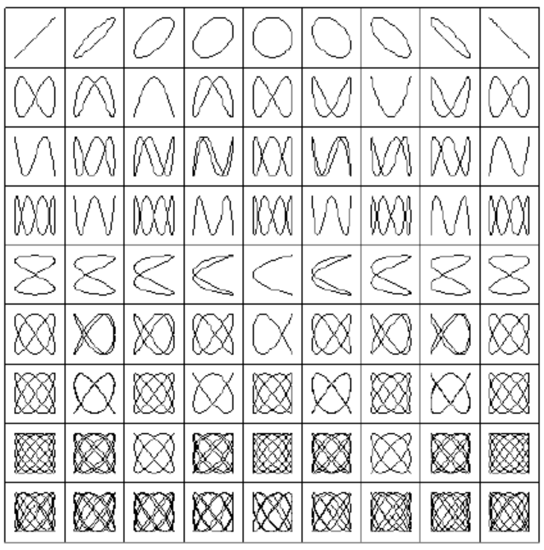

Fig. 1

Interpreting What We See

Now: to the art. **Figures 1** and **3** show the patterns. If the two frequencies are the same, there will be a simple figure, the shape of which depends upon the phase shift between them. If they are perfectly in phase with each other there will be a straight line at a 45° angle (**see Figure 3-a**). If the two frequencies are not in phase, the line becomes an oval then a circle when the phase shift is at 90° (**see Figure 3-c**). At 180° out of phase the oval again becomes a straight line at 45° but now slanting in the opposite direction (**see Figure 3-e**).

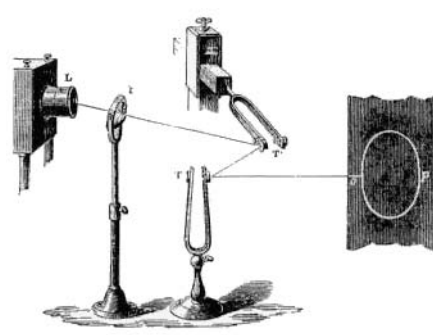

Fig. 2

If the frequencies are not an exact simple ratio of each other (but close), the Lissajous figure will appear to "move," slowly changing orientation as the phase angle between the two waveforms rolls between 0° and 180°. If the two frequencies are locked in an exact ratio between each other, the Lissajous figure will remain stable on the screen.

All right, so maybe beautiful music and beautiful shapes aren't quite as simple as stated at the beginning of this article. But I still remember the first time I saw a Lissajous figure on an oscilloscope and how I was struck by its mesmerizing beauty as it appeared to rotate like a coin spinning slowly on a counter top. You can see this for yourself at *www.people.nnov.ru/fractal/Lissa.htm*. You may never "hear" harmonies quite the same again. So listen with your ears, and your eyes and, as always, stay tuned.

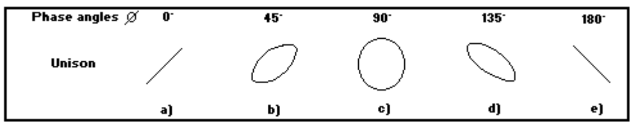

Fig. 3

Roger Goodman is a musician, mathematician, punster, reader of esoteric books and sometime writer, none of which pays the mortgage. For that, he is a computer network guy for a law firm. He has been part of the Los Angeles old-time & contra-dance music community for over thirty years. While not a dancer, he does play fiddle, guitar, harmonica, mandolin, banjo & spoons. Roger has a penchant for trivia and obscura and sometimes tries to explain how the clock works when asked only for the time. He lives with his wife, Monika White, in Santa Monica.